| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Doubling | Use practical activities using manipultives including cubes and Numicon to demonstrate doubling | Draw pictures to show how to double numbers <br> Double 4 is 8 $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ $\square$ | Partition a number and then double each part before recombining it back together. |
| Counting in multiples | Count the groups as children are skip counting, children may use their fingers as they are skip counting. | Children make representations to show counting in multiples. | Count in multiples of a number aloud. <br> Write sequences with multiples of numbers. $2,4,6,8,10$ $5,10,15,20,25,30$ |
| Making equal groups and counting the total | Use manipulatives to create equal groups. | Draw to show $2 \times 3=6$ <br> Draw and make representations | $2 \times 4=8$ |


|  <br> Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Repeated addition | Use different objects to add equal groups | Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? | Write addition sentences to describe objects and pictures. |
| Understanding arrays | Use objects laid out in arrays to find the answers to 2 lots 5,3 lots of 2 etc. | Draw representations of arrays to show understandino | $\begin{gathered} 3 \times 2=6 \\ 2 \times 5=10 \end{gathered}$ |
|  |  |  |  |



| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Multiplication is commutative | Create arrays using counters and cubes and <br> Numicon. <br> Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. | Use representations of arrays to show different calculations and explore commutativity. | $\begin{aligned} & 12=3 \times 4 \\ & 12=4 \times 3 \\ & \left\lvert\, \begin{array}{l} \begin{array}{l} \text { Use an array to write } \\ \text { multiplication sentences and } \\ \text { reinforce repeated addition. } \end{array} \\ \\ \\ \\ \\ 5+5+5=15 \\ 3+3+3+3+3=15 \\ 5 \times 3=15 \\ 3 \times 5=15 \end{array}\right. \end{aligned}$ |
| Using the Inverse <br> This should be taught alongside division, so pupils learn how they work alongside each other. |  |  | $\begin{aligned} & 2 \times 4=8 \\ & 4 \times 2=8 \\ & 8 \div 2=4 \\ & 8 \div 4=2 \\ & 8=2 \times 4 \\ & 8=4 \times 2 \\ & 2=8 \div 4 \\ & 4=8 \div 2 \end{aligned}$ <br> Show all 8 related fact family sentences. |









| Objective \& Strategy | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Division as grouping | Use cubes, counters, objects or place value counters to aid understanding. <br> 24 divided into groups of $6=4$ $96 \div 3=32$ | Continue to use bar modelling to aid solving division problems. $\begin{aligned} & 20 \div 5=? \\ & 5 \times ?=20 \end{aligned}$ | How many groups of 6 in $\begin{gathered} 24 ? \\ 24 \div 6=4 \end{gathered}$ |
| Division with arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created. $\begin{array}{rl} \operatorname{Eg} 15 \div 3=5 & 5 \times 3=15 \\ 15 \div 5=3 & 3 \times 5=15 \end{array}$ | Draw an array and use lines to split the array into groups to make multiplication and division sentences | Find the inverse of multiplication and division sentences by creating eight linking number sentences. $\begin{aligned} & 7 \times 4=28 \\ & 4 \times 7=28 \\ & 28 \div 7=4 \\ & 28 \div 4=7 \end{aligned}$ $28=7 \times 4$ $28=4 \times 7$ $4=28 \div 7$ $7=28 \div 4$ |




## Long Division

Step 1-a remainder in the ones

## $h$ t o <br> 041 R1 <br> $4 \longdiv { 1 6 5 }$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).
4 goes into 16 four times.
4 goes into 5 once, leaving a remainder of 1 .
th $\mathrm{h} t \mathrm{o}$
$8 \longdiv { 0 4 0 0 R 7 }$
8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds $(3,200)$.
8 goes into 32 four times $(3,200 \div 8=400)$
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7 .

## Long Division

Step 1 continued...


When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4=4$, write that four under the 7 , and subract. This finds us the remainder of 3 .

Check: $4 \times 61+3=247$

$$
\begin{array}{r}
\text { th h to } \\
0402 \\
\hline \begin{array}{r}
1609 \\
\frac{-8}{1}
\end{array}
\end{array}
$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4=8$, write that eight under the 9 , and subract. This finds us the remainder of 1 .

Check: $4 \times 402+1=1,609$

## Long Division

Step 2-a remainder in the tens

| 1. Divide. | 2. Multiply \& subtract. | 3. Drop down the next digit. |
| :---: | :---: | :---: |
| $\begin{array}{r} t 0 \\ 2 \longdiv { 2 8 } \end{array}$ <br> Two goes into 5 two times, or 5 tens $\div 2=2$ whole tens - - but there is a remainder! | $\begin{gathered} t 0 \\ 2 \\ 2 \longdiv { 5 8 } \\ \frac{-4}{1} \end{gathered}$ <br> To find it, multiply $2 \times 2=4$, write that 4 under the five, and subtract to find the remainder of 1 ten. | $\begin{array}{r} t \circ \\ 29 \\ 2 \longdiv { 5 8 } \\ -4 \downarrow \\ \hline 18 \end{array}$ <br> Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18. |


| 1. Divide. | 2. Multiply \& subtract. | 3. Drop down the next digit. |
| :---: | :---: | :---: |
| $\begin{array}{r} t \circ \\ 29 \\ 2 \longdiv { 5 8 } \\ =-4 \\ \hline 18 \end{array}$ <br> Divide 2 into 18. Place 9 into the quotient. | $\begin{array}{r} t \circ \\ 29 \\ 2 \longdiv { 5 8 } \\ \frac{-4}{18} \\ -18 \\ \hline 0 \end{array}$ <br> Multiply $9 \times 2=18$, write that 18 under the 18 , and subtract. | $\begin{array}{r} t \circ \\ 29 \\ 2 \longdiv { 5 8 } \\ \frac{-4}{18} \\ -18 \\ \hline 0 \end{array}$ <br> The division is over since there are no more digits in the dividend. The quotient is 29 . |

## Long Division



